A Comparison of Kriging and Cokriging for Estimation of Underwater Acoustic Communication Performance

George P. Kontoudis and Daniel J. Stilwell

Bradley Department of Electrical and Computer Engineering
Center for Marine Autonomy and Robotics
Virginia Tech, Blacksburg, USA

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VIRGINIA TECH

Motivation

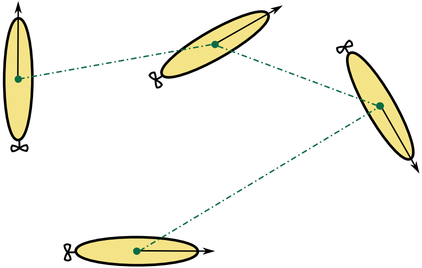
Setup: Multiple underwater vehicles

Motivation: Collaborative autonomy with AUVs

Kriging = Gaussian process

Objective: Build acoustic communication performance maps in real time

Question: Does kriging works well or model-based multivariate kriging (cokriging) works better?



Problem Description

Given:

- Identical measurement model for two agents
- Signal-to-noise ratio (SNR) measurements from an approximate communication performance model
- Range measurements at every communication event

Goals:

- Predict the underwater acoustic communication performance
- Compute the variance of the prediction

Steps:

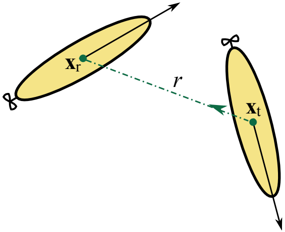
1. Use ordinary kriging (univariate approach)
2. Use multicollocated cokriging (multivariate approach) → proposed methodology
3. Compare the methodologies

Communication Performance

- Identical measurement model of all agents,

$$Y_i(\mathbf{x};t) = Z(\mathbf{x};t) + \epsilon$$

  - $Y_i(\mathbf{x};t)$: measurement of communication performance
  - $Z(\mathbf{x};t)$: Gaussian random field
  - $\epsilon \sim (0, \sigma_{\mathrm{Y}}^2)$: zero-mean Gaussian noise
- Acoustic communication performance is the SNR
- Higher SNR results in better transmitted signal
- Employ the passive sonar equation



An acoustic communication scenario

Passive Sonar Model

The passive sonar equation is expressed,

$$\mathrm{SNR} = \mathrm{SL} - \mathrm{TL} - \mathrm{NL} + \mathrm{DI}$$

- SL: source level - manufacturer
- TL: transmission loss
- NL: noise level
- DI: directivity index - assume negligible

$$\mathrm{TL}(r) = \mathrm{TL_{sph}}(r) - \mathrm{TL_a}(r) = 20 \log r - 0.00556r$$

- $\mathrm{TL_{sph}}$: spherical spreading loss - spherical spreading
- $\mathrm{TL_a}$: attenuation - frequency $f = 25$ kHz, absorption coefficient $a = 5.56$
- $r = \|\mathbf{x}_\mathrm{r} - \mathbf{x}_\mathrm{t}\|_2$: range of two vehicles

Noise Level

The noise comprises of ambient noise, transient noise, and self-noise,

$$\mathrm{NL} = \mathrm{NL}_{\mathrm{amb}} + \mathrm{NL}_{\mathrm{trans}} + \mathrm{NL}_{\mathrm{self}}$$

- $\mathrm{NL}_{\mathrm{amb}}$: ambient noise
  - $\mathrm{NL}_{\mathrm{amb}} = \mathrm{NL}_{\mathrm{ship}} \oplus \mathrm{NL}_{\mathrm{SS}} = \mathrm{NL}_{\mathrm{SS}}$
  - $\mathrm{NL}_{\mathrm{ship}}$: shipping noise - Wenz curves
  - $\mathrm{NL}_{\mathrm{SS}}$: sea state noise - approximated by the Wenz curves
  - $\mathrm{NL}_{\mathrm{SS}} \gg \mathrm{NL}_{\mathrm{ship}}$ for $f = 25$ kHz
- $\mathrm{NL}_{\mathrm{trans}}$: transient noise (e.g. biological) - negligible for high signal frequency
- $\mathrm{NL}_{\mathrm{self}}$: self-noise (e.g. propeller cavitation) - negligible for high signal frequency

The simplified communication performance model,

$$\boxed{\mathrm{SNR} = \mathrm{SL} - 20 \log r + 0.00556 r - \mathrm{NL}_{\mathrm{SS}}}$$

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logical) - neg

cavitation) -

Propeller radiated noise - ignored

Part of the self noise, but we are interested in isotropic sensing

Doppler effect - ignored

We are interested in applications with relatively slow-moving AUVs

The simplified communication performance model,

$$\boxed{\mathrm{SNR} = \mathrm{SL} - 20 \log r + 0.00556 r - \mathrm{NL}_{\mathrm{SS}}}$$

Ordinary Kriging - Problem Setup

The Gaussian random field is modeled as,

$$Z(\mathbf{x}) = \mu + \nu(\mathbf{x}),$$

- μ: unknown constant mean - large scale variation
- $\nu(\mathbf{x})$: zero-mean Gaussian random field - medium scale variation

Assumption

$Z(\mathbf{x}) \in \mathbb{R}$: second-order stationary random field

Employ a linear unbiased estimator,

$$\boxed{\hat{Z}(\mathbf{x}_0) = \sum_{j=1}^{N_j} \beta_j Z(\mathbf{x}_j) + (1 - \sum_{j=1}^{N_j} \beta_j)\mu = \boldsymbol{\beta}^\intercal \mathbf{Z}(\mathbf{x})}$$

- $\boldsymbol{\beta} = [\beta_1 \ \ldots \ \beta_{N_j}]^\intercal$: unknown weights
- $\sum_{j=1}^{N_j} \beta_j = 1$: relaxes the assumption of a known global mean - unbiased estimator

Ordinary Kriging - Minimization

Formulate the unconstrained minimization problem with a Lagrange multiplier,

$$\boldsymbol{\beta}_{\mathrm{OK}} = \boldsymbol{\Gamma}_{\mathrm{OK}}^{-1}\boldsymbol{\gamma}_{\mathrm{OK}}$$

- $\boldsymbol{\beta}_{\mathrm{OK}} = [\boldsymbol{\beta}^{\intercal}\ \lambda_{\mathrm{OK}}]^{\intercal}$: vector of unknown weights
- λ_{OK}: Lagrange multiplier

The non-singular matrix $\boldsymbol{\Gamma}_{\mathrm{OK}} := \begin{bmatrix} \boldsymbol{\Gamma} & \mathbf{1} \\ \mathbf{1}^{\intercal} & 0 \end{bmatrix}$ considers the redundancy of measurements

The vector $\boldsymbol{\gamma}_{\mathrm{OK}} := \begin{bmatrix} \boldsymbol{\gamma}_0 \\ 1 \end{bmatrix}$ takes into account the closeness of the measurements to $\mathbf{x}_0$

Ordinary Kriging - Unique Solution

The unique solution,

$$\boldsymbol{\beta} = \boldsymbol{\Gamma}^{-1}\left(\boldsymbol{\gamma}_0 - \mathbf{1}\lambda_{\mathrm{OK}}\right)$$

where the Lagrange multiplier,

$$\lambda_{\mathrm{OK}} = \frac{\mathbf{1}^\mathsf{T}\boldsymbol{\Gamma}^{-1}\boldsymbol{\gamma}_0 - 1}{\mathbf{1}^\mathsf{T}\boldsymbol{\Gamma}^{-1}\mathbf{1}}$$

The ordinary kriging variance,

$$\sigma^2_{\mathrm{OK}}(Z(\mathbf{x}_0)) = \mathrm{Var}_{\mathrm{OK}}\{Z(\mathbf{x}_0)\} = \boldsymbol{\beta}^\mathsf{T}\boldsymbol{\gamma}_0 + \lambda_{\mathrm{OK}}$$

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Disadvantage - Univariate approach
Use only the SNR measurements w/o considering the range of vehicles

Multivariate Spatial Estimation

Q: How can we use model knowledge to reinforce the estimation process?

The simplified communication performance model,

$$\boxed{\mathrm{SNR}(r) = \mathrm{SL} - 20\log r + 0.00556r - \mathrm{NL}_{\mathrm{SS}}}$$

Use range of vehicles r measurements alongside SNR measurements in the estimation process!

Ordinary Cokriging - Problem Setup

Key Idea: Augments the estimation process with the covariances and cross-covariances of the variables involved in the process.

Application: Use the range of the vehicles as a secondary variable in cokriging in order to improve the SNR estimation.

The ordinary cokriging estimator for two variables,

$$\hat{Z}(\mathbf{x}_0) = \sum_{j=1}^{N_j} \beta_{j,1} Z_1(\mathbf{x}_j) + \sum_{l=1}^{N_l} \beta_{l,2} Z_2(\mathbf{x}_l) = \boldsymbol{\beta}_{\mathrm{COK},1}^{\intercal} \mathbf{Z}_1(\mathbf{x}) + \boldsymbol{\beta}_{\mathrm{COK},2}^{\intercal} \mathbf{Z}_2(\mathbf{x})$$

where the solution to the minimization problem,

$$\boldsymbol{\beta}_{\mathrm{COK}} = \boldsymbol{\Gamma}_{\mathrm{COK}}^{-1} \boldsymbol{\gamma}_{\mathrm{COK}}$$

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Practical Challenges
1. Modeling of all covariances and cross-covariances
2. All covariances and cross covariances jointly need to be positive definite
3. Solution generates very large linear systems, $(N_j + N_l + 2)$-equations

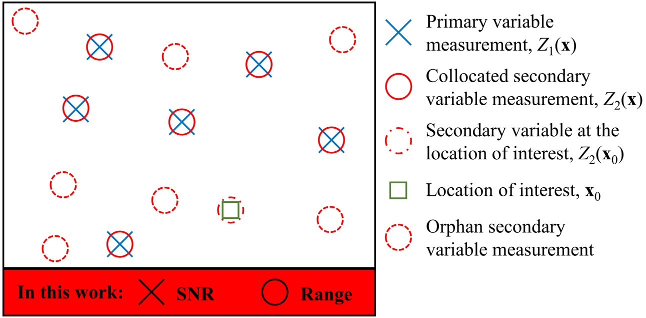
where the solution to the minimization problem,

$$\beta_{\mathrm{COK}} = \mathbf{\Gamma}_{\mathrm{COK}}^{-1} \gamma_{\mathrm{COK}}$$

Multicollocated Ordinary Cokriging - Problem Setup

Multicollocated cokriging accounts for

1. All SNR measurements
2. All range measurements at the locations of the SNR measurements
3. Range at the location of interest



Lemma

The multicollocated cokriging model (or Markov Model 2) has been proven to be necessary and sufficient for cokriging in the stationary case.

Proof.

The proof follows from[1] □

[1] Andre G Journel, 1999, Markov models for cross-covariances, *Mathematical Geology*.

Multicollocated Ordinary Cokriging - Preliminaries

Assumption (Markov Screening)

The primary variable Z_1 at any location $\mathbf{x}_1$ depends conditionally only on the secondary variable Z_2 at location $\mathbf{x}_1$,

$$\mathrm{E}\{Z_1(\mathbf{x}_1) \mid Z_2(\mathbf{x}_1), Z_2(\mathbf{x}_2)\} = \mathrm{E}\{Z_1(\mathbf{x}_1) \mid Z_2(\mathbf{x}_1)\}.$$

Assumption (Bayesian Updating)

The primary and the secondary variables are linearly related through the correlation coefficient $\rho_{12}(0)$ at any location,

$$\mathrm{E}\{Z_1(\mathbf{x}) \mid Z_2(\mathbf{x})\} = \rho_{12}(0) Z_2(\mathbf{x}).$$

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Variables
Primaray variable Z_1: SNR
Secondary variable Z_2: range

Practically
Linear-log relation of Z_1 with Z_2
Expect more smooth results

Multicollocated Ordinary Cokriging - Problem Formulation

The covariogram,

$$\gamma_{12}(\mathbf{h}) = p\gamma_2(\mathbf{h})$$

- $p = \rho_{12}(0)\sigma_1/\sigma_2$: slope of the linear regression
- σ_1: standard deviations of the primary variable
- σ_2: standard deviations of the secondary variable

Regression model of the primary variable on the secondary variable,

$$R(\mathbf{x}) = Z_1(\mathbf{x}) - pZ_2(\mathbf{x})$$

- $R(\mathbf{x})$: orthogonal residual
- Since $Z_1(\mathbf{x})$ and $Z_2(\mathbf{x})$ are Gaussian, $R(\mathbf{x})$ is also Gaussian

Multicollocated Ordinary Cokriging - Unique Solution

The orthogonal residual can be computed with the ordinary kriging,

$$\hat{R}(\mathbf{x}_0) = \boldsymbol{\beta}_{\mathrm{R}}^{\intercal} R(\mathbf{x}),$$

- $\boldsymbol{\beta}_{\mathrm{R}}$: residual corresponding weights of the ordinary kriging

Multicollocated ordinary cokriging estimator for two variables yields,

$$\boxed{\hat{Z}_1(\mathbf{x}_0) = pZ_2(\mathbf{x}_0) + \hat{R}(\mathbf{x}_0) = \sum_{j=1}^{N_j} \beta_{\mathrm{R},j} Z_{1,j} + p\left(Z_2(\mathbf{x}_0) - \sum_{l=1}^{N_l-1} \beta_{\mathrm{R},l} Z_{2,l}\right)}$$

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- $\boldsymbol{\beta}_{\mathrm{R}}$: residua

Advantages
1. Does not require the cross-covariance function
2. Significantly smaller system of equations
2. $(N_j + N_l + 2)$-equations, $N_l > N_j \rightarrow (N_j + 1)$-equations

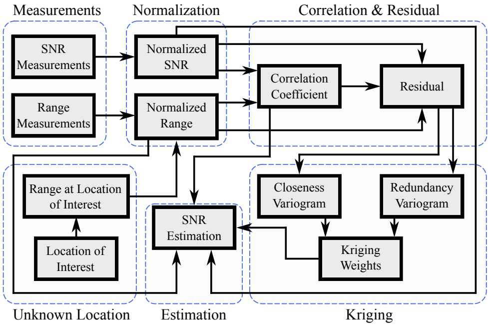
Multicollocated

$$\boxed{\hat{Z}_1(\mathbf{x}_0) = pZ_2(\mathbf{x}_0) + \hat{R}(\mathbf{x}_0) = \sum_{j=1}^{N_j} \beta_{\mathrm{R},j} Z_{1,j} + p\left(Z_2(\mathbf{x}_0) - \sum_{l=1}^{N_l-1} \beta_{\mathrm{R},l} Z_{2,l}\right)}$$

Estimation Structure

The structure incorporates six stages,

1. Collection of measurements
2. Normalization of measurements
3. Computation of the correlation coefficient and the orthogonal residual
4. Ordinary kriging of the residual
5. Unknown location
6. Estimation the communication performance



The normalization follows,

$$\tilde{Z}_{\delta,j} = \frac{Z_{\delta,j} - \mu_\delta}{\sqrt{\mathrm{Var}\{Z_\delta\}}}$$

Semivariogram

We model the semivariogram as a spherical function,

$$\gamma(h) = \begin{cases} C_1(0)\left(\frac{3}{2}\frac{h}{\alpha} - \frac{1}{2}\left(\frac{h}{\alpha}\right)^3\right) & , \ h < \alpha \\ C_1(0) & , \ h \geq \alpha \end{cases}$$

- α: kriging range - beyond α, measurements are considered uncorrelated
- h: distance of the measurements
- $C_1(0)$: sill - in practice $C_1(0) = 1$ for the normalized data

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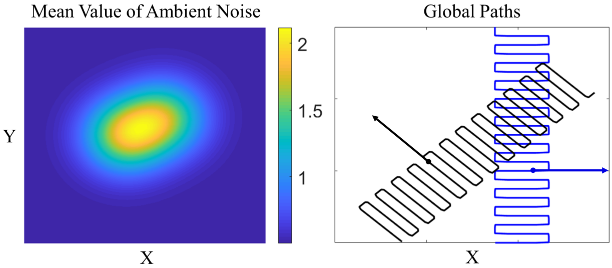
Estimation of Parameters
The semivariogram parameters are user defined in this work
In practice they should be experimentally identified

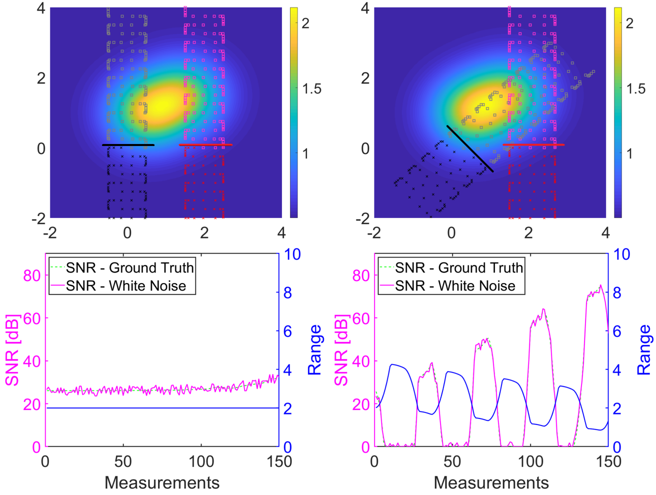
Simulation Environment

- The latent underlying mean of ambient noise follows $\mu_{\mathrm{amb}}(\mathbf{x}) = 0.3 + 1.2\mathrm{e}^{-\|\mathbf{x}-[0.5\ \ 1]^\intercal\|^2} + \mathrm{e}^{-\|\mathbf{x}-[1.5\ \ 1.5]^\intercal\|^2}$
- Higher mean values represent more corrupted SNR with noise
- Signal frequency $f = 25$ kHz
- Resulting mean $\mu_{\mathrm{amb}}(\mathbf{x}) \in [0.50, 2.12]$ corresponds to $\mathrm{NL}_{\mathrm{amb}} \in [25, 45]$ dB
- Extreme environment, ranges from 1 to 33 knots for wind speed
- Source level $\mathrm{SL} = 181$ dB



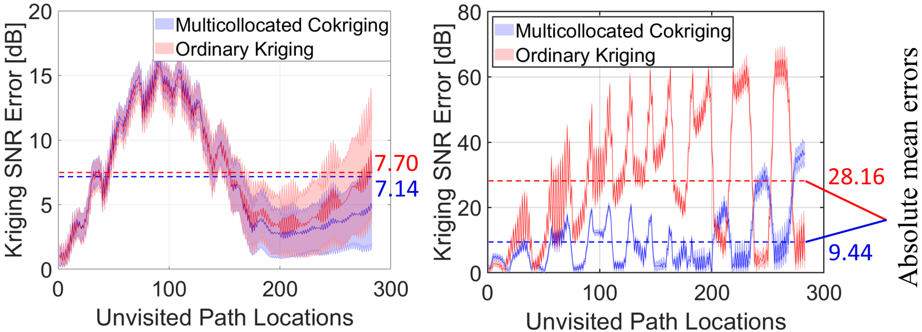
Communication Performance Estimation - First Set

- 150 locations of measurements - black for 1 and red for 2
- 283 unknown locations of interest - gray for 1 and magenta for 2
- Did not collect measurements from increased ambient noise area
- SNR and range measurements are provided in the bottom row
- Cases:
  1. Correlation coefficient $\rho_{12}(0) = -0.098$
  2. Correlation coefficient $\rho_{12}(0) = -0.993$



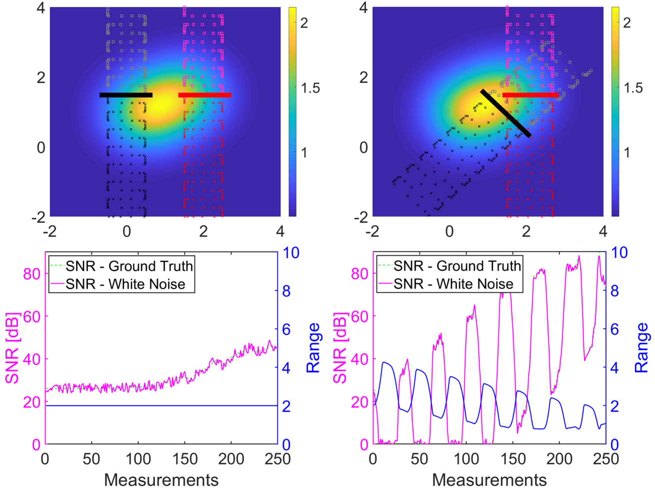
Comparison - First Set

- Absolute error of the SNR with OK [- -]
- Absolute error of the SNR with MCOK [- -]
- First case, OK and MCOK have identical estimation outcomes
- Second case, MCOK outperforms and its mean is significantly lower 66.47%



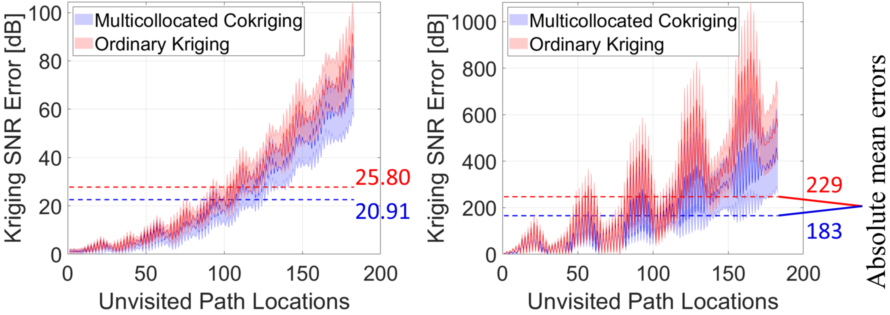
Communication Performance Estimation - Second Set

- 250 locations of measurements - black for 1 and red for 2
- 183 unknown locations of interest - gray for 1 and magenta for 2
- Collect measurements from the area with increased ambient noise
- SNR and range measurements are provided in the bottom row
- Cases:
    1. Correlation coefficient $\rho_{12}(0) = -0.064$
    2. Correlation coefficient $\rho_{12}(0) = -0.957$



Comparison - Second Set

- Absolute error of the SNR with OK [- -]
- Absolute error of the SNR with MCOK [- -]
- Second set, insufficient results for both techniques, even with more measurements
- First case, MCOK produces lower mean error 18.71%
- Second case, COK produces significantly lower mean 32.92%

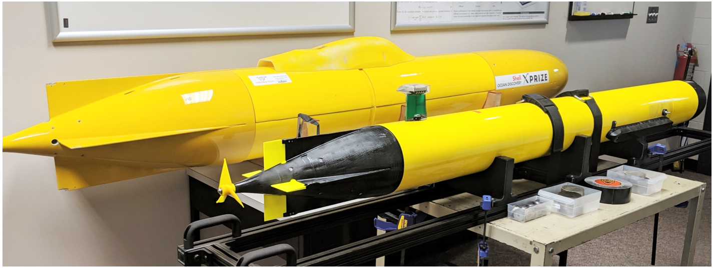


Conclusions

- Illustrate deficiencies in kriging for generating SNR estimates
- Using range as a secondary variable in a cokriging formulation outperforms kriging
- Overall, the proposed multivariate framework outperforms the univariate approach
- Only in certain cases the ordinary kriging computes similar absolute errors
- In realistic applications:
    1. Assumption of stationary global mean for both techniques is rather conservative
    2. Semivariogram parameters should be experimentally estimated
    3. Assumption of linear relationship for primary and secondary variables should be dropped

Future Work

- Formulating online, distributed communication performance estimation algorithm
- Incorporate anisotropic sensing
- Employ universal kriging techniques to capture trend variations
- Estimate semivariogram parameters with maximum likelihood techniques
- Application with our 690-AUVs (we are currently building 4)
- Envision to predict online the communication performance in a distributed fashion



Thank You!